Questionnaire “Logic and Computability”

Summer Term 2024

Contents

11 Temporal Logic

11.1 Temporal Logic

11.1.1 Give the definition of a *Kripke structure*. Explain the components of the tuple a Kripke structure consists of. Give an example of a Kripke structure in the representation of a graph.

Solution

There is no solution available for this question yet.

11.1.2 Give the definition of *paths* and *words* of Kripke structures. Give an example in which you draw a graph representing a Kripke structure, and give one possible infinite path and corresponding word.

Solution

There is no solution available for this question yet.

11.1.3 What does a *computation tree* of a Kripke structure represent? Give an example in which you draw a graph representing a Kripke structure, and draw the first 3 levels of the computation tree of this Kripke structure.

Solution

There is no solution available for this question yet.

11.1.4 The temporal operators describe properties that hold along a given infinite path ρ through the computation tree of a Kripke structure. Given two formulas φ and ψ describing state properties.

- Which are the properties that ρ needs to satisfy such that $\rho \models G\varphi$?
- Which are the properties that ρ needs to satisfy such that $\rho \models F\varphi$?
- Which are the properties that ρ needs to satisfy such that $\rho \models X\varphi$?
- Which are the properties that ρ needs to satisfy such that $\rho \models \varphi U \psi$?

Solution

There is no solution available for this question yet.

11.1.5 Give the definition of the syntax of the computation tree logic $CTL^\star$. In particular, give the definition of state formulas and path formulas.

Solution

There is no solution available for this question yet.

11.1.6 Give an intuitive explanation of the semantics of computation tree logic $CTL^\star$. Therefore, explain the semantics of the introduced path quantifiers and temporal operators with respect to the computation tree of a Kripke structure.

Solution

There is no solution available for this question yet.

11.1.7 Translate the following sentences in computation tree logic $CTL^\star$.

- In every execution the system gives a grant infinitely often.
- There exists an execution in which the system sends a request finitely often.

Solution

- The Boolean variable g represents "The system gives a grant."
  $\varphi_1 \coloneqq AGFg$
- The Boolean variable r represents "The system sends a request."
  $\varphi_2 \coloneqq EFG\neg r$

11.1.8 Translate the following sentences in computation tree logic $CTL^\star$.

- For any execution, it always holds that whenever the robot visits region A, it visits region C within the next two steps.
- There exists an execution such that the robot visits region C within the next two steps after visiting region A.

Solution

We use the following Boolean variables:

- a represents "The robot visits region A"
- b represents "The robot visits region B"
- c represents "The robot visits region C"
- $\varphi_1 \coloneqq AG(a \rightarrow Xc \vee XXc)$
- $\varphi_2 \coloneqq EG(a \rightarrow Xc \vee XXc)$

11.1.9 Translate the following sentences in computation tree logic $CTL^\star$.

- The robot can visit region A infinitely often and region C infinitely often
- Always, the robot visits region A infinitely often and region C infinitely often.
- If the robot visits region A infinitely often, it should also visit region C finitely often.

Solution

We use the following Boolean variables:

- a represents "The robot visits region A"
- b represents "The robot visits region B"
- c represents "The robot visits region C"
- $\varphi_1 \coloneqq E(GFa \wedge GFc)$
- $\varphi_2 \coloneqq A(GFa \wedge GFc)$
- $\varphi_3 \coloneqq A(GFa \rightarrow FG\neg c)$

11.1.10 Given the following execution word w of a Kripke structure. Evaluate the formula φ on w. Evaluate each sub-formula for any execution step using the provided table.

- $w = \{\}, \{a\}, \{a\}, \{b\}, \{\}, \{a\}, \{a, b\}^\omega$
- $\varphi = Xa \lor aUb$

Step	0	1	2	3	4	5	ω
a	0	1	1	0	0	1	1
b	0	0	0	1	0	0	1
Xa	1	1	0	0	1	1	1
aUb	0	1	1	1	0	1	1
$Xa \lor aUb$	1	1	1	1	1	1	1

11.1.11 Given the following execution word w of a Kripke structure. Evaluate the formula φ on w. Evaluate each sub-formula for any execution step using the provided table.

- $w = \{\}, \{a\}, \{\}, \{a, b, c\}, \{a\}, \{a, b\}, (\{a\}, \{a, c\}, \{a, c\})^\omega$
- $\varphi = Ga \to (Fb \lor c)$

Step	0	1	2	3	4	5	ω		
a	0	1	0	1	1	1	1	1	1
b	0	0	0	1	0	1	0	0	0
c	0	0	0	1	0	0	0	1	1
Ga	0	1	0	1	1	1	1	1	1
Fb	0	1	0	1	1	1	0	0	0
$Fb \lor c$	0	1	0	1	1	1	0	1	1
$Ga \to (Fb \lor c)$	0	1	0	1	1	1	0	1	1

11.1.12 Given the following execution word w of a Kripke structure. Evaluate the formula φ on w. Evaluate each sub-formula for any execution step using the provided table.

- $w = \{\}, \{a\}, \{\}, \{a, b, c\}, \{a\}, \{a, b\}, (\{a\}, \{a, c\}, \{a, c\})^\omega$
- $\varphi = GFa \to (FG\neg b \land c)$

Step	0	1	2	3	4	5	ω		
a	0	1	0	1	1	1	1	1	1
b	0	0	0	1	0	1	0	0	0
c	0	0	0	1	0	0	0	1	1
GFa	1	1	1	1	1	1	1	1	1
$FG\neg b$	1	1	1	1	1	1	1	1	1
$FG\neg b \land c$	0	0	0	1	0	0	0	1	1
$GFa \to (FG\neg b \land c)$	0	0	0	1	0	0	0	1	1

11.1.13 Given the following execution word w of a Kripke structure. Evaluate the formula φ on w. Evaluate each sub-formula for any execution step using the provided table.

- $w = \{\}, \{a\}, \{\}, \{a, b\}, \{a\}, \{a, b\}, (\{a\}, \{a, b\}, \{a\})^\omega$
- $\varphi = FGa \to FGb$

Step	0	1	2	3	4	5	ω		
a	0	1	0	1	1	1	1	1	1
b	0	0	0	1	0	1	0	1	0
FGa									
FGb									
$FGa \to FGb$									

11.1.14 Given the following execution word w of a Kripke structure. Evaluate the formula φ on w. Evaluate each sub-formula for any execution step using the provided table.

- $w = \{a\}, \{a\}, \{a\}, \{b, c\}, \{a\}, \{a, b\}(\{a\}, \{c\})^\omega$
- $\varphi = aUc \vee Fb$

Step	0	1	2	3	4	5	ω	
a	1	1	1	0	1	1	1	0
b	0	0	0	1	0	1	0	0
c	0	0	0	1	0	0	0	1
aUc								
Fb								
$aUc \vee Fb$								

11.1.15 Given the following Kripke structure $\mathcal{K}$. Does the initial state s_0 of $\mathcal{K}$ satisfy the following formulas?

- $\varphi_1 := EXX(a \wedge b)$
- $\varphi_2 := EXAX(a \wedge b)$

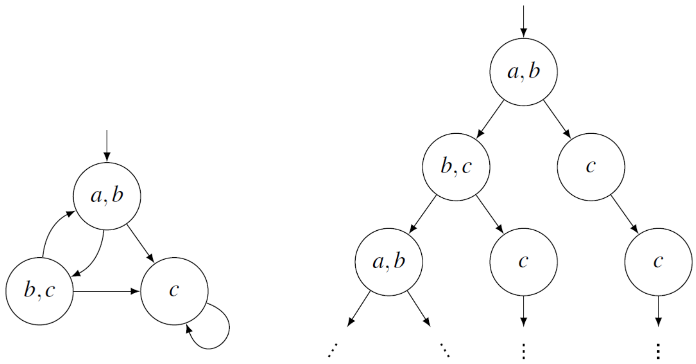

Figure 1: Left: Kripke structure of Example 7, Right: Corresponding computation tree

Solution

- $s_0 \models EXX(a \wedge b)$
- $s_0 \not\models EXAX(a \wedge b)$

11.1.16 Given the following Kripke structure $\mathcal{K}$. Does the initial state s_0 of $\mathcal{K}$ satisfy the following formulas?

- $\varphi_1 := EXp$
- $\varphi_2 := EG\neg p$

Solution

- $s_0 \models EXp$
- $s_0 \models EG\neg p$

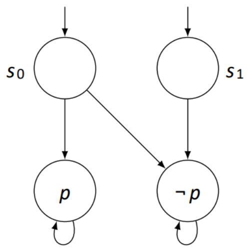

Figure 2: Kripke structure of Example 8

11.1.17 Consider an ordinary traffic junction with incoming lanes from the north, south, east and west. We want to formulate relevant constraints that a traffic light system has to fulfill.

Give a set of propositional variables that model whether the north and south *or* the east and the west get the

- green,
- yellow or
- red

light, respectively.

Formulate the following sentences using $CTL^{\star}$:

(a) The north/south lanes will never get the green light at same time as the east/west lanes.

(b) Whenever the north/south lane receive the green light it will stay green until it changes to yellow.

(c) When the east/west lane has the red light, it will eventually get the yellow and red light until the light switches to green.

Solution

There is no solution available for this question yet.