Questionnaire “Logic and Computability”

Summer Term 2024

Contents

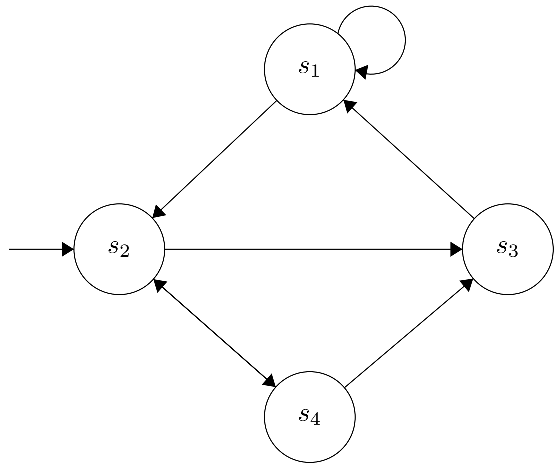
10 Symbolic Encoding

10.1 Transition Systems

10.1.1 Draw the graph for the *transition system* $\mathcal{T}$ with:

- $S = \{s_1, s_2, s_3, s_4\}$,
- $S_0 = \{s_2\}$,
- $R = \{(s_1, s_2), (s_1, s_1), (s_2, s_4), (s_2, s_3), (s_3, s_1), (s_4, s_2), (s_4, s_3)\}$,

Solution



10.1.2 Consider the example of an elevator. Initially, the elevator is in the ground floor. From the ground floor, it can either go basement, stay there for a while, and then go back to the ground floor, or it can go from the ground floor to the second floor, stay there for a while, and go back to the ground floor. While traveling between ground floor to second floor, the elevator passes the first floor, but it cannot stop there.

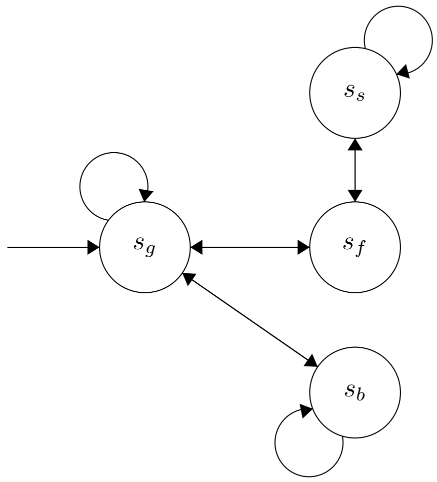
Model this elevator as *transition system*.

Solution

We use the following states:

- s_g indicates that the elevator is on the ground floor.
- s_b indicates that the elevator is in the basement.
- s_s indicates that the elevator is on the second floor.
- s_f indicates that the elevator is passing the first floor.

The transition system is then given by: $\mathcal{T} = (S,\, S_0,\, R)$ with $S = \{s_g, s_b, s_s, s_f\}$, $S_0 = \{s_g\}$, $R = \{(s_g, s_g), (s_g, s_b), (s_b, s_b), (s_b, s_g), (s_g, s_f), (s_f, s_s), (s_s, s_s), (s_s, s_f), (s_f, s_g)\}$



10.1.3 Consider the example of a controller for a lamp.

Initially the light is off. Pressing the button once turns on the light and the light glows white. From this state, any short-lasting pressure of the button causes the light to switch its color randomly between white, red, green, blue, and yellow. At any state, pressing the button for a longer time turns the light off.

Model the lamp controller as *transition system*.

Solution

There is no solution available for this question yet.

10.1.4 Define the *transition system* from the following symbolically encoded transition relations and draw the corresponding graph:

$$\begin{gathered}(v_1 \land v_0 \land \neg v_1' \land \neg v_0')\ \lor \\ (\neg v_1 \land v_0 \land \neg v_1' \land v_0')\ \lor \\ (v_1 \land v_0 \land v_1' \land v_0')\end{gathered}$$

Solution

There is no solution available for this question yet.

10.1.5 Define the *transition system* from the following symbolically encoded transition relations and draw the corresponding graph:

$$(\neg v_1 \land \neg v_0 \land v_1' \land v_0') \lor$$
$$(\neg v_1 \land v_0 \land \neg v_1' \land \neg v_0') \lor$$
$$(\neg v_1 \land \neg v_0 \land \neg v_1' \land \neg v_0')$$

Solution

There is no solution available for this question yet.

10.2 Symbolic Encoding

10.2.1 Given a state space of size $|S| = 2^4 = 16$, give the symbolic encoding for the following states: (a) s_7, (b) s_{15}, and (c) s_{10}.

Solution

For the symbolic encoding we need 4 Boolean variables, $\{v_3, \ldots, v_0\}$. Let v_3 be the most significant bit, and v_0 the least significant bit.

(a) $s_7 = \neg v_3 \land v_2 \land v_1 \land v_0$
(b) $s_{15} = v_3 \land v_2 \land v_1 \land v_0$
(c) $s_{10} = v_3 \land \neg v_2 \land v_1 \land \neg v_0$

10.2.2 Given is the set of states $S = \{s_0, \ldots, s_7\}$. Find formulas in propositional logic that symbolically represent the sets $A = \{s_7, s_6, s_3, s_2\}$, $B = \{s_1, s_3, s_5, s_7\}$, and $C = \{s_7, s_6, s_0, s_1\}$.

Solution

$A = \{s_7, s_6, s_3, s_2\} = (v_2 \land v_1 \land v_0) \lor (v_2 \land v_1 \land \neg v_0) \lor (\neg v_2 \land v_1 \land v_0) \lor (\neg v_2 \land v_1 \land \neg v_0)$
$= v_1$

$B = \{s_1, s_3, s_5, s_7\} = (\neg v_2 \land \neg v_1 \land v_0) \lor (\neg v_2 \land v_1 \land v_0) \lor (v_2 \land \neg v_1 \land v_0) \lor (v_2 \land v_1 \land v_0)$
$= v_0$

$C = \{s_7, s_6, s_0, s_1\} = (v_2 \land v_1 \land v_0) \lor (v_2 \land v_1 \land \neg v_0) \lor (\neg v_2 \land \neg v_1 \land \neg v_0) \lor (\neg v_2 \land \neg v_1 \land v_0)$
$= (v_2 \land v_1) \lor (\neg v_2 \land \neg v_1)$

10.2.3 Find a symbolic binary encoding for $X = \{0, 1, \ldots, 31\}$. Use it to find formulas that symbolically represent the sets A and B and simplify the formulas:

- $A = \{12, 13, 14, 15, 28, 29, 30, 31\}$
- $B = \{x \in X \mid 0 \leq x \leq 15\}$

Furthermore, give the formulas representing the sets $C = A \cap B$ and $D = A \cup B$.

Solution

We use 5 Boolean variables, $\{v_4, \ldots, v_0\}$, for the encoding.

$A = (v_2 \land v_3)$
$B = \neg v_4$

10.2.4 Given a state space of size $|S| = 2^4 = 16$. Give the symbolic encoding for the following states: (a) s_4, (b) s_9, and (c) s_{13}.

Solution

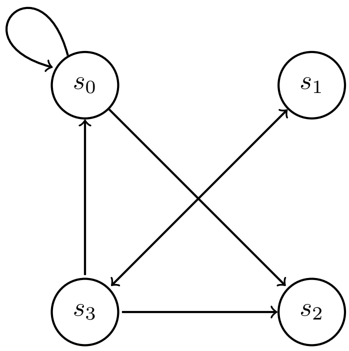
There is no solution available for this question yet.

10.2.5 Given is the set of states $S = \{s_0, \ldots, s_7\}$. Find formulas in propositional logic that symbolically represent the sets $A = \{s_0, s_2, s_4, s_6\}$, $B = \{s_0, s_1, s_2, s_3\}$, and $C = \{s_7, s_1\}$.

Solution

There is no solution available for this question yet.

10.2.6 Find a *symbolic encoding* for the *transition relation* of the following *transition system* and simplify your formulas. Use a binary encoding to encode the states, e.g., encode the state s_2 with the formula $v_1 \land \neg v_0$.



Solution

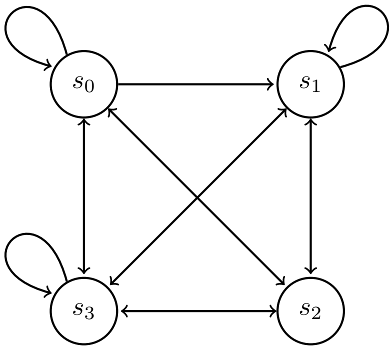
Using the variables v_1 and v_0, we can define the transition relation using the following formula:

$$\neg v_1 \land \neg v_0 \land (\neg v_1' \land \neg v_0' \lor v_1' \land \neg v_0') \lor$$
$$\neg v_1 \land v_0 \land v_1' \land v_0' \lor$$
$$v_1 \land v_0 \land (\neg v_1' \land v_0' \lor \neg v_1' \land \neg v_0' \lor v_1' \land \neg v_0')$$

We can further simplify the formula to:

$$\neg v_1 \land \neg v_0 \land \neg v_0' \lor$$
$$\neg v_1 \land v_0 \land v_1' \land v_0' \lor$$
$$v_1 \land v_0 \land (\neg v_1' \land v_0' \lor \neg v_0')$$

10.2.7 Find a *symbolic encoding* for the *transition relation* of the following *transition system* and simplify your formulas. Use a binary encoding to encode the states, e.g., encode the state s_2 with the formula $v_1 \land \neg v_0$.



Solution

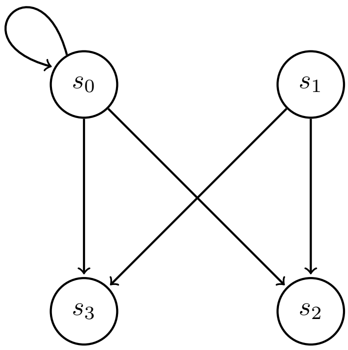
Using the variables v_1 and v_0, we can define the transition relation using the following formula:

$$\begin{gathered}\neg v_1 \wedge \neg v_0 \wedge (\neg v_1' \wedge \neg v_0' \vee \neg v_1' \wedge v_0' \vee v_1' \wedge \neg v_0' \vee v_1' \wedge v_0') \vee \\ \neg v_1 \wedge v_0 \wedge (\neg v_1' \wedge v_0' \vee v_1' \wedge \neg v_0' \vee v_1' \wedge v_0') \vee \\ v_1 \wedge \neg v_0 \wedge (\neg v_1' \wedge \neg v_0' \vee \neg v_1' \wedge v_0' \vee v_1' \wedge v_0') \vee \\ v_1 \wedge v_0 \wedge (\neg v_1' \wedge \neg v_0' \vee \neg v_1' \wedge v_0' \vee v_1' \wedge \neg v_0' \vee v_1' \wedge v_0')\end{gathered}$$

We can further simplify the formula to:

$$\begin{gathered}\neg v_1 \wedge \neg v_0 \vee \\ \neg v_1 \wedge v_0 \wedge (v_0' \vee v_1' \wedge \neg v_0') \vee \\ v_1 \wedge \neg v_0 \wedge (\neg v_1' \vee v_1' \wedge v_0') \vee \\ v_1 \wedge v_0\end{gathered}$$

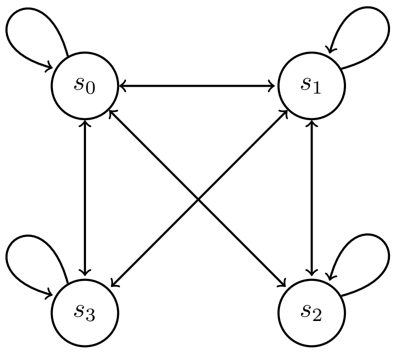
10.2.8 Find a *symbolic encoding* for the set of initial states and the *transition relation* of the following *transition system* and simplify your formulas. Use a binary encoding to encode the states, e.g., encode the state s_2 with the formula $v1 \wedge \neg v_0$.



Solution

There is no solution available for this question yet.

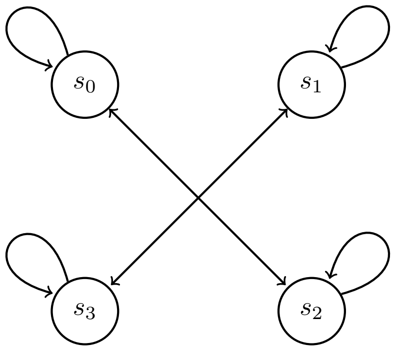
10.2.9 Find a *symbolic encoding* for the set of initial states and the *transition relation* of the following *transition system* and simplify your formulas. Use a binary encoding to encode the states, e.g., encode the state s_2 with the formula $v1 \wedge \neg v_0$.



Solution

There is no solution available for this question yet.

10.2.10 Find a *symbolic encoding* for the set of initial states and the *transition relation* of the following *transition system* and simplify your formulas. Use a binary encoding to encode the states, e.g., encode the state s_2 with the formula $v1 \wedge \neg v_0$.



Solution

There is no solution available for this question yet.

10.2.11 What is the main advantage of symbolic set operations over non-symbolic operations that enumerate all set elements explicitly?

Solution

There is no solution available for this question yet.

10.2.12 Given a state space of size $|S| = 2048$, find a symbolic binary encoding for this state space and compute the characteristic function for the sets of states

$$B = \{s_0, s_1, s_2, ..., s_{1023}\} \text{ and } C = \{s_{512}, s_{513}, , s_{514}, ..., s_{1535}.\}$$

Then compute the characteristic function for the sets $D = B \cup C$ and $E = B \setminus C$. If possible, simplify the formulas.

Solution

There is no solution available for this question yet.

10.2.13 Find a symbolic binary encoding for $X = \{0, 1, \ldots, 31\}$. Use it to compute formulas in propositional logic that symbolically represent the following sets.

- $B = \{4, 5, 12, 13, 20, 21, 28, 29\}$
- $C = \{1, 2, 13, 14\}$

Compute the characteristic functions of the following sets by symbolic operations, using your results from before.

(a) $D = B \cup C$
(b) $E = X \setminus D$

Solution

There is no solution available for this question yet.

10.2.14 Find a symbolic binary encoding for $X = \{0, 1, \ldots, 31\}$. Use it to compute formulas in propositional logic that symbolically represent the following sets.

- $B = \{x \in X \mid \text{x is even}\}$
- $C = \{x \in X \mid \text{x is odd}\}$
- $D = \{0, 1, 2, 3, 4, 5, 6, 7\}$

Compute the characteristic functions of the following sets by symbolic operations, using your results from before.

(a) $E = B \cup D$
(b) $F = C \cap E$
(c) $G = E \setminus F$

Solution

There is no solution available for this question yet.

10.2.15 Find a symbolic binary encoding for $X = \{0, 1, \ldots, 31\}$. Use it to compute formulas in propositional logic that symbolically represent the following sets.

- $B = \{8, 9, 10, 11, 12, 13, 14, 15\}$
- $C = \{x \in X \mid 0 \leq x \leq 15\}$

Compute the characteristic functions of the following sets by symbolic operations, using your results from before.

(a) $D = B \cup C$
(b) $E = B \cap C$
(c) $F = C \setminus B$

Solution

There is no solution available for this question yet.

10.2.16 Assume you are given the formulas a and b, which symbolically represent the sets A and B, respectively. Give the formula c, which symbolically represents the set $C = A \setminus B$.

Solution

There is no solution available for this question yet.

10.2.17 Assume you are given the formulas a and b, which symbolically represent the sets A and B, respectively. What would you have to check on a, b to test whether or not A is a strict subset of B, i.e., $A \subset B$?

Solution

There is no solution available for this question yet.

10.2.18 Given a state space of size $|S| = 64$. Find a symbolic binary encoding for this state space and compute the formulas that symbolically represent the sets

$$B = \{s_{32}, s_{33}, s_{34}, ..., s_{63}\} \text{ and } C = \{s_{16}, s_{17}, , s_{18}, ..., s_{40}\}.$$

Following, compute the formulas that represent the sets $D = B \cap C$, $E = B \cup C$, $F = B \setminus C$ and $G = C \setminus B$.

Solution

There is no solution available for this question yet.

10.2.19 Given a state space of size $|S| = 64$, find a symbolic binary encoding for this state space and compute the formulas that symbolically represent the sets of states

$$B = \{s_{16}, s_{17}, s_{18}, ..., s_{32}\} \text{ and } C = \{s_{24}, s_{25}, , s_{26}, ..., s_{64}.\}$$

Then compute the formulas that symbolically represent the sets $D = B \cap C$ and $E = B \cup C$.

Solution

There is no solution available for this question yet.

10.2.20 Listed are the participants of a seminar as well as their choice of snacks. Find a symbolic encodings for the participants. For for this encoding, give the symbolic representation of the set B of all participants that ordered *bananas*, and the set C of all participants that ordered cake.

Name	Snack
Alice	banana
Bob	cake
Carl	banana
David	banana
Eve	cake
Frank	cake
Greg	orange
Hank	cake

Solution

There is no solution available for this question yet.

10.2.21 Listed are the participants of a seminar as well as their choice of snacks. Find a symbolic encodings for the participants. For for this encoding, give the symbolic representation of the set B of all participants that ordered *bananas*, and the set C of all participants that ordered cake.

Name	Snack
Alice	banana
Bob	cake
Carl	banana
David	banana
Eve	cake
Frank	cake
Greg	orange
Hank	cake

Solution

There is no solution available for this question yet.

10.2.22 The following table shows eight students and their means of transportation. Find a symbolic encodings representing the list of students. For this encoding, give the symbolic representation of the set B of all students that go by *bike*, and the set C of all students that go by *car*.

Name	Transportation
Alice	Car
Bob	Bike
Carl	Tram
David	Bike
Eve	Tram
Frank	Bike
Greg	Tram
Hank	Bike

Solution

There is no solution available for this question yet.

10.2.23 Consider the domain $A = \{Spain, France, Italy, Germany\}$ and the two different symbolic encodings for A given below. Which one gives a shorter symbolic representation for the set $B = \{France, Italy\}$? Illustrate your answer by giving the representing formulas for B in both encodings.

Encoding 1

Element	v_1	v_0
Spain	0	0
France	1	0
Italy	0	1
Germany	1	1

Encoding 2

Element	v_1	v_0
Spain	0	0
France	1	0
Italy	1	1
Germany	0	1

Solution

There is no solution available for this question yet.

10.2.24 Consider the domain $A = \{Spain, France, Italy, Germany\}$ and the two different symbolic encodings for A given below. Which one gives a shorter symbolic representation for the set $B = \{France, Germany\}$? Illustrate your answer by giving the representing formulas for B in both encodings.

Encoding 1

Element	v_1	v_0
Spain	0	0
France	1	0
Italy	0	1
Germany	1	1

Encoding 2

Element	v_1	v_0
Spain	0	0
France	1	0
Italy	1	1
Germany	0	1

Solution

Using encoding 1, we end up in the following formula:
$b = v_1$

Using encoding 2, we end up in the following formula:
$b = (v_1 \land \neg v_0) \lor (\neg v_1 \land v_0)$

Encoding 1 gives a shorter symbolic representation for the set $B = \{France, Germany\}$.