Binary Splitting

Recursive algorithm:

$$S_{[n_1,n_2)} = \sum_{n=n_1}^{n_2-1} \frac{a(n)}{b(n)} \frac{p(n_1)\cdots p(n)}{q(n_1)\cdots q(n)}$$

Compute $P = p(n_1)\cdots p(n_2-1)$, $Q = q(n_1)\cdots q(n_2-1)$, $B = b(n_1)\cdots b(n_2-1)$ and T with

$$S_{[n_1,n_2)} = \frac{T}{B\cdot Q}$$

$n_2 - n_1 < 4 \quad \to$ directly

$n_2 - n_1 \geq 4 \quad \to$ split

$$\begin{gathered} P = P_L \cdot P_R \\ Q = Q_L \cdot Q_R \\ B = B_L \cdot B_R \\ T = B_R \cdot Q_R \cdot T_L + B_L \cdot P_L \cdot T_R \end{gathered}$$