Main Result

$$S = \sum_{n=0}^{\infty} \frac{a(n)}{b(n)} \, \frac{p(0) \cdots p(n)}{q(0) \cdots q(n)}$$

with $a(n)$, $b(n)$, $p(n)$, $q(n)$ small integers $= O(n^c)$

can be computed to N digits in time

$$O\left(M(N)\,(\log N)^2\right) = O\left(N\,(\log N)^3\,\log\log N\right).$$